

Duality of a Supersymmetric Standard Model

Nobuhiro Maekawa ²

*Department of Physics, Kyoto University,
Kyoto 606-01, Japan*

Abstract

We examine a dual theory of a Supersymmetric Standard Model(SSM) in terms of an $SU(3)_C$ gauge group. In this scenario, it is naturally understood that at least one quark (the top quark) should be heavy, i.e., almost the same order as the weak scale. Moreover, the supersymmetric Higgs mass parameter μ can naturally be expected to be small. This model automatically induces nine pairs of composite Higgs fields, which may be observed in the near future.

²e-mail: maekawa@gauge.scphys.kyoto-u.ac.jp

Recently, it has become clear that certain aspects of four dimensional supersymmetric field theories can be analyzed exactly [1, 2, 3, 4]. One of the most interesting aspects is “duality” [1]. By using “duality”, we can infer the low energy effective theory of a strong coupling gauge theory. Does nature use this “duality”? In this paper, we would like to discuss a duality of a Supersymmetric Standard Model(SSM).

First we would like to review Seiberg’s duality. Following his discussion [1], we examine $SU(N_C)$ supersymmetric (SUSY) QCD with N_F flavors of chiral superfields,

	$SU(N_C)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_R$
Q^i	N_C	N_F	1	1	$(N_F - N_C)/N_F$
\bar{Q}_j	\bar{N}_C	1	\bar{N}_F	-1	$(N_F - N_C)/N_F$

which has the global symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)_B \times U(1)_R$. This theory is called the electric theory. In the following, we would like to take $N_F \geq N_C + 2$, though in the case $N_F \leq N_C + 1$ there are a lot of interesting features [3, 5, 6, 7]. Seiberg suggests [1] that in the case $N_F \geq N_C + 2$ at the low energy scale the above theory is equivalent to the following $SU(\tilde{N}_C)$ SUSY QCD theory ($\tilde{N}_C = N_F - N_C$) with N_F flavors of chiral superfields q_i and \bar{q}^j and meson fields T_j^i ,

	$SU(\tilde{N}_C)$	$SU(N_F)_L$	$SU(N_F)_R$	$U(1)_B$	$U(1)_R$
q_i	\tilde{N}_C	\tilde{N}_F	1	$N_C/(N_F - N_C)$	N_C/N_F
\bar{q}^j	$\tilde{\bar{N}}_C$	1	N_F	$-N_C/(N_F - N_C)$	N_C/N_F
T_j^i	1	N_F	\tilde{N}_F	0	$2(N_F - N_C)/N_F$

and with a superpotential

$$W = q_i T_j^i \bar{q}^j. \quad (1)$$

This theory is called the magnetic theory. The above two theories satisfy the ’t Hooft anomaly matching conditions [8]. Moreover Seiberg showed that they are consistent with the decoupling theorem [9]. Namely, if we introduce a mass term only for superfields Q^{N_F} and \bar{Q}_{N_F}

$$W = m Q^{N_F} \bar{Q}_{N_F} \quad (2)$$

in the original (electric) theory, the dual (magnetic) theory has vacuum expectation values(VEVs) $\langle q \rangle = \langle \bar{q} \rangle = \sqrt{m}$ and $SU(N_f - N_c)$ is broken to $SU(N_f - N_c - 1)$, which is consistent with the decoupling of the heavy quark in the original theory.

Next, we would like to discuss a duality of a SUSY Standard Model (SSM). We introduce ordinary matter superfields

$$\begin{aligned} Q_L^i &= (U_L^i, D_L^i) : (3, 2)_{\frac{1}{6}}, & U_{Ri}^c &: (\bar{3}, 1)_{-\frac{2}{3}}, & D_{Ri}^c &: (\bar{3}, 1)_{\frac{1}{3}} \\ L^i &= (N_L^i, E_L^i) : (1, 2)_{-\frac{1}{2}}, & E_{Ri}^c &: (1, 1)_1, & i &= 1, 2, 3, \end{aligned} \quad (3)$$

which transform under the gauge group $SU(3)_{\bar{C}} \times SU(2)_L \times U(1)_Y$. There are no Higgs superfields. We would like to examine the magnetic theory of this electric theory with respect to the gauge group $SU(3)_{\bar{C}}$. In the following, we neglect the lepton sector for simplicity. Since $N_F = 6$, the dual gauge group is also $SU(3)_C$ ($\tilde{N}_C = N_F - N_C$), which we would like to assign to the QCD gauge group. A subgroup, $SU(2)_L \times U(1)_Y$, of the global symmetry group $SU(6)_L \times SU(6)_R \times U(1)_B \times U(1)_R$ is gauged. When we assign $Q = (U_L^1, D_L^1, U_L^2, D_L^2, U_L^3, D_L^3)$ and $\bar{Q} = (U_R^{c1}, D_R^{c1}, U_R^{c2}, D_R^{c2}, U_R^{c3}, D_R^{c3})$, the $SU(2)_L$ generators are given by

$$I_L^a = I_{L1}^a + I_{L2}^a + I_{L3}^a, \quad a = 1, 2, 3, \quad (4)$$

where I_{Li}^a are generators of $SU(2)_{Li}$ symmetries which rotate (U_L^i, D_L^i) , and the generator of hypercharge Y is given by

$$Y = \frac{1}{6}B - (I_{R1}^3 + I_{R2}^3 + I_{R3}^3), \quad (5)$$

where I_{Ri}^a are generators of $SU(2)_{Ri}$ symmetries which rotate (U_{Ri}^c, D_{Ri}^c) . In this theory, the global symmetry group is $SU(3)_{QL} \times SU(3)_{UR} \times SU(3)_{DR} \times U(1)_B \times U(1)_R$. Then we can write down the quantum numbers of dual fields;

$$\begin{aligned} q_{Li} &= (d_{Li}, -u_{Li}) : (3, \bar{2})_{\frac{1}{6}}, & u_R^{ci} &: (\bar{3}, 1)_{-\frac{2}{3}}, & d_R^{ci} &: (\bar{3}, 1)_{\frac{1}{3}} \\ M_j^i &: (1, 2)_{-\frac{1}{2}}, & N_j^i &: (1, 2)_{\frac{1}{2}} \end{aligned} \quad (6)$$

under the standard gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Here $M_j^i \sim Q_L^i U_{Rj}^c$ and $N_j^i \sim Q_L^i D_{Rj}^c$ are the meson fields and we assign $q = (d_L^1, -u_L^1, d_L^2, -u_L^2, d_L^3, -u_L^3)$ and $\bar{q} = (d_R^{c1}, -u_R^{c1}, d_R^{c2}, -u_R^{c2}, d_R^{c3}, -u_R^{c3})$. It is interesting that the matter contents of both theories are almost the same. The difference is the existence of nine pairs of Higgs superfields M_j^i and N_j^i and their Yukawa terms coupling to ordinary matter superfields,

$$W = -y_u q_L^i N_j^j u_{Rj}^c + y_d q_L^i M_j^j d_{Rj}^c. \quad (7)$$

Here neglecting the effect of $U(1)_Y$ gives $y_u = y_d$. The Yukawa couplings can be expected to be of order one because of the strong dynamics³.

In the following, we only assume that the duality can be realized even if SUSY breaking terms

$$\mathcal{L}_{SB}^e = \sum_{i=1}^3 \left(m_{Q_i}^2 |Q^i|^2 + m_{U_i}^2 |U_i|^2 + m_{D_i}^2 |D_i|^2 \right) + \frac{1}{2} \sum_{a=1,2,3} \mu_a \lambda_a \lambda_a, \quad (8)$$

where λ_a are gauginos, are introduced and that the scale Λ at which the duality becomes a bad description is higher than the SUSY breaking scale. We will discuss these assumptions later. In general we can take $m_1^2 \geq m_2^2 \geq m_3^2$. The global symmetry is broken to $U(1)$ ⁸ by these SUSY breaking terms. By using the perturbation [10] we can get the SUSY breaking terms of the dual theory

$$\begin{aligned} \mathcal{L}_{SB}^m &= \sum_{i=1}^3 \left(m_{q_i}^2 |q_i|^2 + m_{u_i}^2 |u^i|^2 + m_{d_i}^2 |d^i|^2 \right) + \sum_{i,j=1}^3 \left(m_{M_{ij}}^2 |M_i^j|^2 + m_{N_{ij}}^2 |N_i^j|^2 \right) \\ &\quad + \frac{1}{2} \sum_{a=1,2,3} \mu_a \lambda_a \lambda_a, \end{aligned} \quad (9)$$

$$m_{q_i}^2 \sim m_{Q_i}^2, \quad m_{u_i}^2 \sim m_{D_i}^2, \quad m_{d_i}^2 \sim m_{U_i}^2, \quad m_{M_{ij}}^2 \sim m_{Q_i}^2 + m_{U_j}^2, \quad m_{N_{ij}}^2 \sim m_{Q_i}^2 + m_{D_j}^2.$$

The Higgs can have a vacuum expectation value (VEV) radiatively [11]. By using the renormalization group equations

$$\frac{d}{dt} m_{N_{ij}}^2 = \frac{1}{8\pi^2} \left(\tilde{N}_C y_u^2 (m_{N_{ij}}^2 + m_{q_i}^2 + m_{u_j}^2) + \frac{1}{2} g_1^2 \text{tr}(Y m^2) - 3g_2^2 \mu_2^2 - g_1^2 \mu_1^2 \right), \quad (10)$$

$$\frac{d}{dt} m_{M_{ij}}^2 = \frac{1}{8\pi^2} \left(\tilde{N}_C y_d^2 (m_{M_{ij}}^2 + m_{q_i}^2 + m_{d_j}^2) - \frac{1}{2} g_1^2 \text{tr}(Y m^2) - 3g_2^2 \mu_2^2 - g_1^2 \mu_1^2 \right), \quad (11)$$

$$\begin{aligned} \frac{d}{dt} m_{q_i}^2 &= \frac{1}{8\pi^2} \left(3(y_u^2 + y_d^2) m_{q_i}^2 + \sum_j^3 (y_u^2 (m_{N_{ij}}^2 + m_{u_j}^2) + y_d^2 (m_{M_{ij}}^2 + m_{d_j}^2)) \right. \\ &\quad \left. + \frac{1}{6} g_1^2 \text{tr}(Y m^2) - \frac{16}{3} g_3^2 \mu_3^2 - 3g_2^2 \mu_2^2 - \frac{1}{9} g_1^2 \mu_1^2 \right), \end{aligned} \quad (12)$$

$$\frac{d}{dt} m_{u_i}^2 = \frac{1}{8\pi^2} \left(3y_u^2 m_{u_i}^2 + \sum_j^3 y_u^2 (m_{N_{ij}}^2 + m_{u_j}^2) - \frac{2}{3} g_1^2 \text{tr}(Y m^2) - \frac{16}{3} g_3^2 \mu_3^2 - \frac{16}{9} g_1^2 \mu_1^2 \right) 3$$

³ By using the superconformal algebra ($D = 3|R|/2$), the conformal dimension D of the meson fields can be calculated as $3/2$, which is much different from the conformal dimension of free fields. Therefore the meson fields must have some strong interaction. It is natural to regard the Yukawa interaction as the strong one. Though this argument is reliable only at the infra-red fixed point, we expect that the Yukawa couplings remains large even if the coupling is not at the infra-red fixed point.

$$\frac{d}{dt}m_{di}^2 = \frac{1}{8\pi^2} \left(3y_d^2 m_{di}^2 + \sum_j^3 y_d^2 (m_{Mij}^2 + m_{dj}^2) + \frac{1}{3}g_1^2 \text{tr}(Ym^2) - \frac{16}{3}g_3^2 \mu_3^2 - \frac{4}{9}g_1^2 \mu_1^2 \right) \quad (14)$$

we can expect that the smallest mass term at the low energy scale will be m_{M33}^2 or m_{N33}^2 unless $\mu_a^2 \ll m^2$. The tree Higgs potential is

$$\begin{aligned} V_H = & \sum_{ij} m_{Nij}^2 |N_i^j|^2 + m_{Mij}^2 |M_i^j|^2 + \frac{1}{8}g_2^2 \sum_a \left| \sum_{ij} (N_j^{\dagger i} \tau_a N_i^j + M_j^{\dagger i} \tau_a M_i^j) \right|^2 \\ & + \frac{1}{8}g_1^2 \left| \sum_{ij} (N_j^{\dagger i} N_i^j - M_j^{\dagger i} M_i^j) \right|^2. \end{aligned} \quad (15)$$

If only m_{Nij}^2 is negative, we can take $\langle N_{33} \rangle = (v, 0)$. In this case, from the Higgs potential (15) we can find that N_i^j have a tendency to have a VEV with an unbroken electromagnetic interaction $U(1)_{EM}$, on the other hand M_i^j have a tendency to have a VEV breaking the $U(1)_{EM}$. In order to avoid breaking the $U(1)_{EM}$, we introduce the following conditions

$$m_{Ui}^2 - m_{D3}^2 > m_Z^2, \quad i = 1, 2, 3, \quad (16)$$

where m_Z is the mass of the Z boson. Moreover if we introduce the conditions

$$m_{Di}^2 - m_{D3}^2 > m_Z^2, \quad (17)$$

$$m_{Qi}^2 - m_{Q3}^2 > m_Z^2, \quad i = 1, 2, \quad (18)$$

only N_{33} can have a VEV. By the above discussions, we will not claim that only the top quark is naturally heavy. We would like to emphasize that the large Yukawa couplings are naturally understood and it is possible that only the top quark has a large mass by imposing some conditions which are not so unnatural.

It is also interesting that the global symmetry $SU(3)_Q \times SU(3)_{UR} \times SU(3)_{DR}$ forbids the term μMN . Namely, we can understand the smallness of the SUSY Higgs mass μ . Phenomenologically the mass μ should be around the SUSY breaking scale m_{SB} . We do not have a definite explanation for realizing $\mu \sim m_{SB}$. However, we would like to emphasize here that we can naturally introduce the global symmetry which forbids the mass term.

Of course, in this model, if further Higgs fields have some VEVs, massless Nambu-Goldstone bosons appear, because the global $U(1)$ symmetries still exist. Therefore in

order to give masses to the other quarks without massless Nambu-Goldstone bosons, we must break the global $U(1)$ symmetries explicitly. There are some ways to break the symmetries. One possibility is to introduce the Higgs fields and the Yukawa couplings in the electric theory, which can break the global symmetries. As another possibility, if we introduce vector-like fields, we can break the symmetries by the SUSY breaking terms [12]. Even when we do not introduce new fields, we can break all the global $U(1)$ symmetries except $U(1)_B$ by introduce SUSY breaking terms violating R-parity [13].

It is a serious problem that the leptons are massless in this model. There are several mechanisms which can induce the Yukawa couplings of leptons [12, 13]. One possibility is to introduce the Pati-Salam gauge group [12], and another one is to break the R-parity [13]. Here we do not discuss these possibilities further.

Finally, we would like to discuss the assumptions introduced previously. Does duality hold even with SUSY breaking terms? This is still an open question though some people [10, 14] analyze this subject. Since some useful techniques such as holomorphy cannot be used in the analysis with SUSY breaking terms, it is difficult to get a definite answer. Therefore in this paper we only assume that there is a duality in this case.

Where is the scale at which the dual description is broken? Since Seiberg's discussion using the superconformal algebra is effective only in the conformal phase, the duality might exist only in the conformal phase (or close to it). If the duality is effective only in the conformal phase, our model would be invalid because the QCD coupling ($\alpha_s \sim 0.11 - 0.12$) may be too far from the infra-red fixed coupling ($\alpha^* \sim 0.6$). However, since the discussions about 'tHooft anomaly matching conditions and about the consistency with the decoupling are effective even in the non-conformal phase, it is not strange that the duality exists even with the coupling far from the infra-red fixed point. If the both electric and magnetic theories are asymptotically free, the dual description will be broken at the scale at which the gauge coupling is not so far from the infra-red fixed point. This is because the difference will be obvious when the couplings of both theories are small enough to use the perturbations [15]. However, what we would like to emphasize here is the possibility that one of them is not asymptotically free. Since both theories with $3N_C/2 < N_F < 3N_C$ have infra-red fixed points, there is the asymptotically non-free phase. Namely, when the gauge coupling is above the infra-red

fixed coupling, the theory is in the asymptotically non-free phase. If this possibility is correct, the dual description may be realized even with the coupling far from the infra-red fixed point, because both theories cannot have the small couplings simultaneously. In this model, if the electric theory is asymptotically non-free, then the smallness of the QCD coupling in magnetic theory may be realized. Of course we know that the duality is realized only in the low energy scale, because some other massive states will exist (*e.g.*, in the dual theory of the dual theory, meson fields become massive.). However you should notice that the mass scale of the heavy particles which should be decoupled is unknown.

Does the above scenario work even if the QCD scale is lower than the SUSY breaking scale m_{SB} ? Here we will only note that if such a duality persists even with SUSY breaking m_{SB} , the QCD scale, which is defined by the divergence of the gauge coupling, should be smaller than the SUSY breaking scale m_{SB} . The reason is the following. Since the SUSY models have an infra-red fixed point, the gauge coupling does not diverge without SUSY breaking. Under the SUSY breaking scale, the running of the coupling changes and the coupling can diverge at some scale, $\Lambda_{QCD} < m_{SB}$.

In summary, “duality” may be interesting even in the real world. We applied this technique to a Supersymmetric Standard Model. Though this model is not a complete model phenomenologically, there are some interesting features. The Higgs fields are induced as composites, a heavy top quark is naturally understood, and there is a global symmetry which forbids the SUSY Higgs mass terms. The model also predicts nine pairs of Higgs superfields. Since we do not know which theories is connected to the real high energy theory, we believe that the duality gives us very rich possibilities for model building.

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